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THE MATHEMATICAL EXPERTISE OF MECHANICAL ENGINEERS – THE CASE OF MECHANISM DESIGN

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Abstract-*In this contribution we present the results of a project that investigates the mathematical qualifications a mechanical engineer needs for working on practical tasks in his daily life. In particular, we report on the results concerning a mechanism design task where certain machine parts have to be moved in order to realize a cutting activity.*

1. INTRODUCTION

The mathematical education of engineers has two major goals: first, to enable students to understand and use (and maybe develop on their own) mathematical models that are used in the application subjects like Engineering Mechanics or Control Theory; secondly, to provide a mathematical basis for their future professional life. The second goal is far more nebulous because it is much harder to obtain information on the mathematical expertise that is necessary in the daily life of an engineer. This contribution reports on a project that investigates the mathematical expertise of mechanical engineers. We restrict ourselves to "standard tasks", so we do not try to capture the research and development sector since our graduates are normally not employed for such work. In (Alpers, 2006), we reported on the results concerning a typical static construction task using a CAD program. Here, we deal with a typical mechanism design task for an industrial cutting device.

2. METHOD OF INVESTIGATION

Although there are some studies on the mathematical content of workplace activity (cf. Bessot/Ridgway 2000, Gainsburg 2005), there are only very few studies on engineering professions. Kent and Noss (2003) and Gainsburg (2006) investigated civil engineering by visiting construction firms, interviewing engineers and managers and joining engineers in their daily life. Although this gives direct access to practical work, the method is very time-consuming, and for an outsider it is hard to get a good understanding in a short time. Therefore, the research presented in this contribution uses a different method. We hire two students during their last semester who already spent one or two semesters in industry. The students are given "typical" tasks which we identify together with a colleague who teaches machine elements, CAD and FEM and who worked for several years in the car industry. The students are paid for 100 hours of work. They use industrial strength programs that are available at the university. The colleague acts as a mentor similar to a group leader in industry. The students are advised to make notes on their thoughts such that their thinking processes can be investigated. The author interviews the students and lets them explain and demonstrate their tool usage. These sessions are recorded with screen recording software.

Based on these data, the author investigates the mathematical concepts that are used to work effectively and efficiently on the task, in particular those ones which are important to make reasonable use of the tools involved. Since the author has some basic experience with the tools, he is able to re-perform the activities of the students for closer investigation. Particular attention is given to so-called "breakdown situations" (Kent and Noss, 2003) where a seemingly correct use of tools leads to problems or unreasonable results.

Cardella and Atman (2005a,b) apply a somewhat similar approach by investigating the role of mathematics in so-called "capstone design projects" where the students were mainly from Industrial Engineering (but also a few from other study courses). They capture the mathematical thinking activities using the more aggregated categories developed by Schoenfeld (1992) whereas the goal of our investigation consists of getting a deeper insight into the more detailed mathematical activities when using technology.

3. THE TASK: DESIGN OF PART OF A CUTTING DEVICE

The task is a typical mechanism design task: a knife has to be moved up and down for cutting off a part of a foil which also has to be moved forward for a certain length. This mechanism is taken from a real machine which is used for producing halogen lamps.

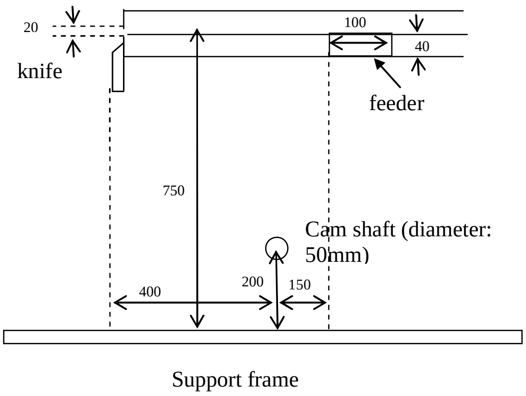

Figure 1. Sketch.

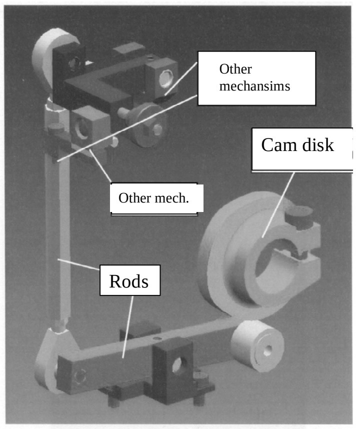

Figure 2. Example.

Figure 1 shows the sketch the students received. It contains the essential dimensions and positions. The students have to construct cam disks (to be placed on the cam shaft) and some connecting rods to generate a mechanism for moving the knife up and down and the feeder back and forth. Since in real engineering life, there are often similar devices from which an engineer gets ideas (instead of developing everything from scratch), we provided them with the example in Figure 2.

Finally, in order to synchronize with other parts of the system, a motion plan is given (Figure 3) which indicates how the motions of the feeder and the knife depend on the rotation angle of the common cam shaft (used by all mechanisms). The plan only prescribes the intervals of constant lift. It is up to the engineer to find functions to get from one constant lift level to the next (e.g. from 80° to 140° in case of the feeder or from 150° to 190° in case of the knife). The linear interpolation is only inserted for optical reasons.

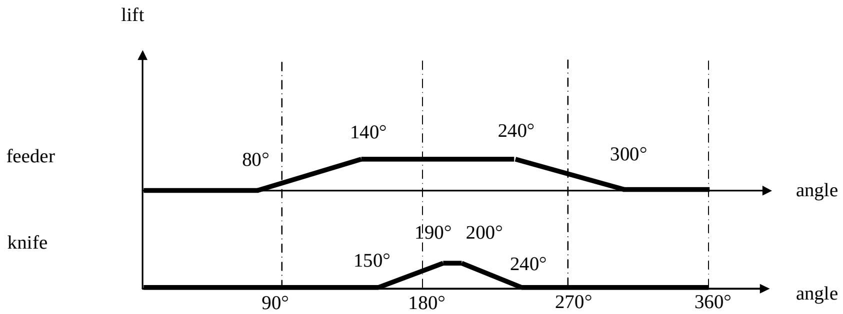

Figure 3. Motion plan.

For the knife, a force of 100N is required to perform the cutting process. For dimensioning the motor driving the cam shaft it is also necessary to compute or estimate the required driving torque.Although there are some studies on the mathematical content of workplace activity (cf. Bessot/Ridgway 2000, Gainsburg 2005), there are only very few studies on engineering professions. Kent and Noss (2003) and Gainsburg (2006) investigated civil engineering by visiting construction firms, interviewing engineers and managers and joining engineers in their daily life. Although this gives direct access to practical work, the method is very time-consuming, and for an outsider it is hard to get a good understanding in a short time. Therefore, the research presented in this contribution uses a different method. We hire two students during their last semester who already spent one or two semesters in industry. The students are given “typical” tasks which we identify together with a colleague who teaches machine elements, CAD and FEM and who worked for several years in the car industry. The students are paid for 100 hours of work. They use industrial strength programs that are available at the university. The colleague acts as a mentor similar to a group leader in industry. The students are advised to make notes on their thoughts such that their thinking processes can be investigated. The author interviews the students and lets them explain and demonstrate their tool usage. These sessions are recorded with screen recording software.

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4. RESULTS AND DISCUSSION

The students knew in principle how to proceed by performing the following steps:

- Design of connecting rods using the CAD program Pro/Engineer®
- Design of motion functions for the knife and the feeder using a tool
- Design of the cam disk using the respective tools in Pro/Engineer®
- Design of the springs which guarantee that the rollers attached to the rods have contact with the cam disk
- Computation of the required driving torque for the motor driving the cam shaft.

Figure 4 depicts the final design results of the two students. We do not report on the geometrical qualifications when using the CAD program Pro/Engineer® because these were already described in (Alpers, 2006).

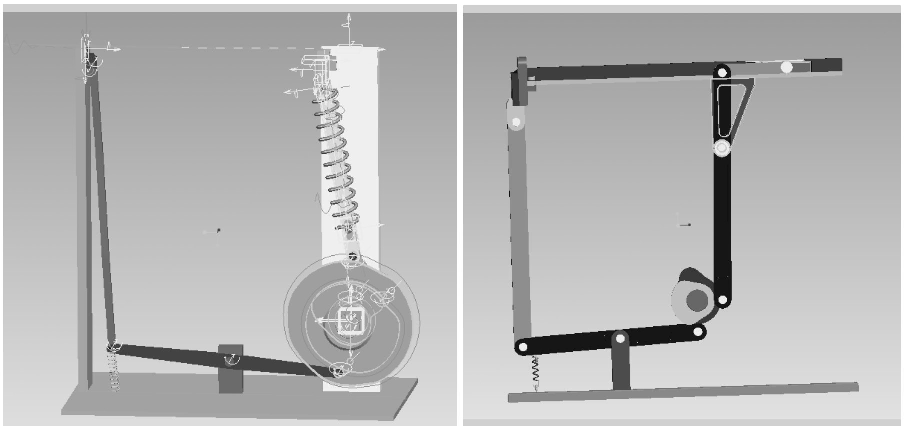

Figure 4. Mechanisms designed by the students

When designing the linkage, one of the students started with a leverage of 1:1 without making any further investigations having in mind that the ratio could be changed later if the required driving torque was too high. The other one made some investigations by hand (with paper and pencil) in order to see how the ratio influences the necessary torque and the shape (largeness) of the cam disk. The student drew the rough shape of the cam disk for some ratios and computed an overly rough estimate for the torque. He realized that moving the swivel joint to the left reduced the necessary torque but makes the cam disk "steeper" and finally chose a ratio of 1:0.8 as a good compromise. According to the colleague involved in the project it is quite common in industry to start with a first guess coming from experience or knowledge of a similar device and make changes only when problems come up later on.

Having constructed the linkage, the students had to construct the "missing parts" of the motion functions for the knife and for the feeder, e.g. for the feeder the function piece on the interval [80°,140°]. For such tasks, the German Association of Engineers (VDI) has set up guidelines where certain functions are proposed, e.g. a polynomial of degree 5 or a modified sine function (see Alpers/Steeb, 1998). There are tools for constructing such functions for a given situation (guaranteeing continuity of the first and second derivative, i.e. velocity and acceleration) and the students had access to such a tool which was programmed in a former diploma thesis. For using the tool the students had to understand its required input and the produced output. The user has to specify the start and end points of the resting phases and the functions to be used between two such phases. Here, functions according to the VDI guideline are offered. The VDI guideline additionally specifies several quality criteria for choosing a function (e.g. maximum absolute value of velocity or acceleration or of the product of these quantities). We told the students to use the modified sine function since this was the guideline for such mechanisms given by the company that uses the cutting mechanism. We will go back to the meaning of the quality criteria below when we discuss a rough model for estimating the necessary torque.

Finally, the tool needs the number of revolutions per minute in order to output the lift over time function in form of a table of values that can be stored in a file.

Such files are the input needed by Pro/Engineer® as motion functions of the knife and the feeder, respectively. Alternatively, Pro/Engineer® offers certain function types to specify the motion (e.g. polynomial, cosine function) and the user must input the parameters occurring in the function type (e.g. coefficients of a polynomial). Finally, the user can also insert piecewise defined functions where the pieces are expressions and the domains are intervals. The user has to take care himself to guarantee continuity with respect to position, velocity and acceleration. We mention this way of input since we also have a tool for producing the motion function according to the VDI guidelines symbolically and we could use this to check the numerical input (cf. remarks below on unwanted numerical effects).

After having specified the motion of the knife and the feeder in Pro/Engineer® by providing the respective files, the CAD program is able to simulate and compute the motion of the centers of the rollers. By constructing a plane which is parallel to the linkage, by rotating this plane with the number of revolutions used earlier on and by projecting the centers of the rollers onto this plane (orthogonally) one gets the motion curves of the centers on this plane. Using the CAD program to construct offset curves, one finally gets the boundary curves of the two cam disks. From this, the disks can be constructed easily by adding a certain depth.

The last design task consists of inserting springs such that the rollers do not leave the boundary of the cam disks. Only the first student had enough time to dimension the spring. He started with a constant value of 100N/m and ended up with 1000N/m when the roller did not leave the disk. He did not perform any optimisation which is quite usual according to the colleague involved.

Finally, for making a dynamic analysis, the force function for the knife had to be specified. Pro/Engineer® allows the same ways of input as in the case of motion functions described above. Only the first student had enough time to perform such an analysis. He specified the force function as being 0 for the period the knife rests and then it goes up to 100N very quickly, remains at this level for a short time before going back to 0. The colleague involved and the author had a different perception of the cutting process: We thought that the cutting takes place when the knife is near its maximum velocity such that one gets a “clear cut”. Like the student, we did not know how the force development in the cutting process really is. The author had a closer look into the force modelling of the student when the resulting maximum driving torque that was computed by Pro/Engineer® did not comply with a computation using a rough model we describe below.

Kent and Noss (2003) observed that mathematical thinking processes, particularly deficits in these processes, can best be investigated in “breakdown situations” where problems come up. We observed three such situations which we report in the sequel.

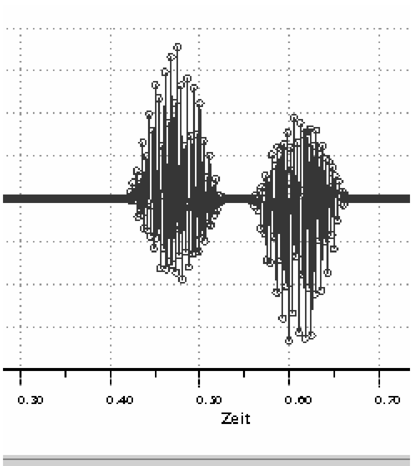

Figure 5. Oscillation in Acceleration.

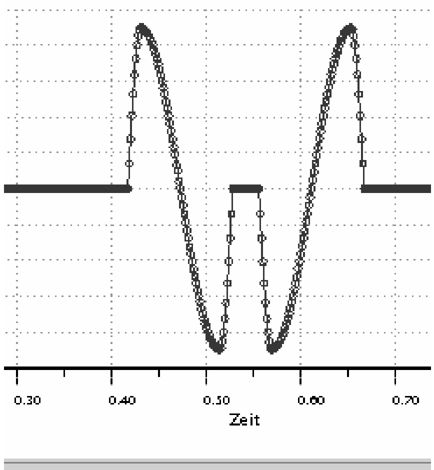

Figure 6. No oscillation.

a) In the analysis of the driving torque performed with Pro/Engineer®, a strange oscillatory behaviour showed up which could not be explained in the given configuration. Since the work within Pro/Engineer® seemed to be correct, we (the author and the student) had a closer look at the input data. The students used the output table of the tool described above as input for describing the motion of the knife in Pro/Engineer®. Pro/Engineer® allows to graph also velocity and acceleration. Figure 5 depicts the acceleration function which already shows a rather “weird” behaviour. It should look like the one shown in figure 6 where the knife has first a positive and then a negative acceleration until its upper rest is reached. It was clear that Figure 5 did not make any sense (although the s(t) function looked quite reasonable). Moreover, extremely high values for acceleration were shown (Figures 5 and 6 have different scales).

A closer inspection of the input data revealed that the tool producing the table of values worked with a fixed output precision of 1/1000 (meters for distance and seconds for time). This was too coarse given a rotational frequency of 60 rpm, i.e. 1 rps, since the data table was constructed using □□=1° which corresponds to □t=0.0028 s. This still produced an acceptable s(t) function (distance over time) but then led to the faulty oscillation behaviour in the derivatives. After the precision of the tool had been extended, the reasonable graph shown in figure 6 was produced.
The situation described above shows how important it is to check the output (here: the driving torque graph) for plausibility. For this, a rough model can be used as will be described below, and the shape must reflect the mechanical situation. For example, when the knife cuts off a piece of the foil the required momentum has its maximum. The situation also showed how easily erroneous results are produced in high-level tools like Pro/Engineer® when wrong or imprecise input is fed in. Therefore, it is very important that the user is able to use all available facilities in order to check the input for reasonability. Here, a closer look at the also available plots of velocity and acceleration revealed the input problems. For interpreting these graphs, the user has to understand the relationships between the graphs (graph of derivative) and hence to know what to expect. The student did not perform these checks on his own but he was able to understand and then perform them after the author had presented them.

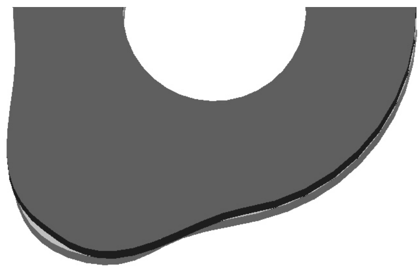

Figure 7. Cam discs.

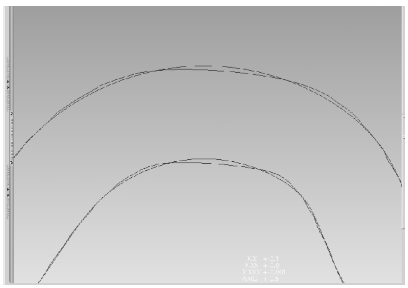

Figure 8. Boundary curves.

b) For checking the cam disk automatically produced by Pro/Engineer® (the light grey one in figure 7) one of the students constructed a cam disk directly in the CAD program by drawing circle sections for the resting phases and using the CAD program for interpolation (the dark grey one in figure 7). This way he recognized that the boundary curve produced automatically by Pro/Engineer® did not have a circle segment at the rest which is not possible, yet he could not find the reason since the input seemed to be okay. Figure 8 depicts that already the motion curve of the roller center shows the same problem, so the problem has nothing to do with the construction of offset curves. This figure also gives an idea for the reason: the common points in both roller center curves are at equidistant disk angles (this is no longer the case after the production of the offset curves!). This led the author to the assumption that both curves might be interpolating the same set of points but differently. An investigation of how Pro/Engineer® produced the curve showed that for rapid production the student chose to compute 25 positions of the mechanism, so

every 14.4° a point is computed and the curve is produced by using spline interpolation. Since the rest interval of the knife has only a length of 10° it is not surprising that the curve produced by Pro/Engineer® deviates from the circle segment shape. Another run using 1000 positions showed no such deviation.
From this breakdown situation one can conclude how important it is to recognize the effects of interpolation. Again, the situation shows how easy an inadequate usage of a high-level tool can lead to unacceptable results, so it is always important to be able do perform simple checks like the student did who compared the automatically constructed disk with a "hands-on approximation" of his own.

c) The author looked up a rough model in literature (Vollmer, 1989) in order to check the results on the required driving torque produced by one of the students. The model uses a power balance: "input power=-(output power + loss)" but the loss is neglected. The input power is the product of the input driving torque Min and the rotational velocity ωin and the output power is the product of the output force Fout and the output velocity vout (we restrict ourselves to the cutting part of the overall mechanism, so vout is the velocity of the knife):

$$M_{in} \cdot \omega_{in} = -F_{out} \cdot v_{out} \quad \text{, hence } M_{in} = \frac{-F_{out} \cdot v_{out}}{\omega_{in}}.$$

The output force is the sum of the applied force (here: the knife force of 100N), the inertial force, the spring force and the gravitational force:

$$F_N(t) \le 100N, \quad F_T = m \cdot a_{out}(t), \quad F_F = c \cdot s_{out}(t)\ , c = 1000N/m, \quad F_S = m \cdot g.$$

Hence:

$$\begin{aligned} & F_{out} \cdot v_{out} = (F_N + F_T + F_F + F_S) \cdot v_{out} \\ & \le 100 \cdot v_{out} + m \cdot a_{out} \cdot v_{out} + 1000 \cdot s_{out} \cdot v_{out} + m \cdot 9.81 \cdot v_{out} \\ & \le 100 \cdot \max\{v_{out}\} + m \cdot \max\{a_{out} \cdot v_{out}\} + 1000 \cdot \max\{s_{out} \cdot v_{out}\} + m \cdot 9.81 \cdot \max\{v_{out}\} \end{aligned}$$

This simple model shows quite well the influence of knife force, velocity and acceleration. The tool used for setting up the motion function already provides the maximum values of the velocity and the product of velocity and acceleration:

$$\max\{v_{out}\} = 0.317[m/s], \ \max\{v_{out} \cdot a_{out}\} = 1.59[m^2/s^3].$$

With a mass of about 0.5 kg and a distance of 0..0.02m (take 0.01m where velocity has its maximum), we get the following upper estimate:

$$100 \cdot 0.317 + 0.5 \cdot 1.59 + 1000 \cdot 0.01 \cdot 0.317 + 0.5 \cdot 9.81 \cdot 0.317 = 31.7 + 0.795 + 3.17 + 1.55 \approx 37.2$$

This shows the dominating influence of the power relating to the knife force. Division by the rotational velocity of 2π [1/s] gives a momentum of about 5.9Nm. The student came up with a momentum of about 3Nm. This has the same order of magnitude but only half the value which gave reason to investigate the causes. We found out that the student had used a knife force function where the force started much earlier and was finished before the velocity reached its maximum (for a clean cut it would be better to have a higher velocity when the cutting process starts). Since the computation presented above is dominated by the product of force and velocity it is quite understandable why the value computed by the student was lower. The model also explains why the company uses a so-called modified sine function for constructing the knife motion because the VDI guideline mentioned above recommends this type of function for getting a low maximum value for the velocity. This shows that simple models like the one given above can have high explanatory value and provide excellent opportunities for performing checks and possibly for detecting hidden assumptions.

5. CONCLUSIONS

The method we used for investigating the mathematical expertise within the daily work of a mechanical engineer proved to be very fruitful. It allowed to probe deeply into the ways standard tools like CAD systems are used for performing practical tasks including the dangers and pitfalls that go with the usage. The colleague who acted as a guide confirmed that most of the work done by the students reflected practical work of junior engineers. He also found the situation quite usual that the students had to work with the mechanism design and analysis part of Pro/Engineer® without having had any introductory course.

The investigation showed that although most of the mathematical concepts and procedures are “buried in technology”, for reasonable usage of the interface mathematical knowledge and understanding is still necessary. In the case of mechanism design, this refers in particular to a good understanding of the different representations of functions and their derivatives as well as of interpolation concepts.

The deeper analysis of breakdown situations (Kent/Noss, 2003) also revealed how easily faulty input or input relying on questionable assumptions produces erroneous analysis results or constructions. For the discovery of faults, it is very important to be able to compare the results with expected behaviour (oscillating torque) and to have instruments of checking at hand like simple constructions (with circle segments) or simple models (power balance). If one does not expect anything one will never see the unexpected. For finding the causes of faults, one should be able to use and interpret all the representation options in the tool (input functions and its derivatives) and also to understand how the tool produces its curves (computation of positions and interpolations). Here, the understanding of possible pitfalls of approximation methods like interpolation is also helpful. For performing variations efficiently, small models like the power model are very useful in order to identify the dominating factors of influence.

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