Relearning mathematics for application classes with computer algebra: The case of control theory

Burkhard Alpers, Department of Mechanical Engineering
Aalen University of Technology and Business, Germany, balper@fh-aalen.de

Abstract
In the second period of study, subsequent to their first industrial internship, students often have forgotten the mathematical concepts and algorithms they learnt during the first year. In this paper, we present an approach and supporting material to encounter this problem by offering mathematical relearning opportunities which are specific to control theory as an example for an application subject relying heavily on mathematical modelling. Using the Computeralgebra-System Maple (TM) as an environment for writing and using the relearning material allows easy experimentation and computation by students and facilitates implementation of tutorial help by instructors.

1 Introduction

In German "Universities for Applied Sciences" which offer compact, practice-oriented studies mainly in engineering and business, mathematics is positioned in the first two (or three) semesters. Many students then spend one practical semester (internship) in industry. Afterwards, they continue with theory like machine dynamics or control theory in mechanical engineering. Often, their mathematical knowledge is no longer available when it is needed in application classes for building mathematical models. When the models are derived, many students lack understanding since they are not familiar with the mathematics used. This way, they are incapable of capturing the correspondence between mathematical properties of the model and application properties. They simply accept the model as is and try to use it by inserting values.
This problem is addressed by providing and implementing an environment for "relearning" mathematics for a certain application area in a notation which is used in the application and with emphasis on those properties of mathematical objects which are important for modelling application behaviour. The application area chosen in the prototype is control theory since it heavily relies on mathematical knowledge (relations between input and output functions, modelling behaviour with functions, laplace transforms etc.). We set up a suite of interconnected computer algebra worksheets which contain all mathematical topics needed in the control theory classes.
The relearning unit is just one part of a larger project which aims at enhancing the interconnection of mathematics and application areas in engineering education. We give a short description of the contents and underlying concepts of this project in section 2. Section 3 contains a description of the mathematical relearning material and additional application-specific material for control theory as well as advised usage scenarios. Accompanying activities which are necessary to make students familiar with the material are outlined in section 4. In section 5 we present the results of an evaluation at the end of the first trial and corresponding adaptations. Finally, we state the main observations and outline future perspectives.

2 Concept for interconnecting mathematics and engineering subjects

The main objective of our project on interconnecting mathematics and engineering is to make students think of mathematics as an integral part of dealing with engineering topics thus avoiding that mathematics is just seen as an intellectual exercise in the first year of studies. To achieve this objective, we make or plan several offerings. During the

first period of study (semesters 1 to 3), we intend to provide learning material for all those subjects which need a lot of mathematics (cf. table 1). The material consists of a documentary hypertext part which contains all results of an application subject with links to underlying mathematical concepts and algorithms such that the links between mathematics and engineering are made explicit and the mathematics needed can be studied "on demand". Moreover, an assignment environment contains all exercises corresponding to the application lectures. It serves to activate students, enables them to check intermediary results and provides tutorial help in case they get stuck or do not find an error. So far, such material is available for the application subject "stress analysis" which is taught in a four-hour lecture. The material is available on the web and a description can be found in [1]. We plan to produce similar material for all subjects of the first period of study listed in table 1.
For those application areas in the second period of study which need many mathematical concepts, we plan to offer mathematical relearning material. This is currently available for control theory and will be described below in more detail. Table 1 also lists other subjects of the second period for which similar material is planned. How to organize relearning effectively is not well-known and there are only a few publications. Glowalla reports in [2] that offering "selective" relearning as opposed to complete relearning of a subject can make relearning more time-efficient. Our approach can be considered as pre-selection according to the needs of a certain application area.

Table 1. Application subjects of concern

1. Period of Study (semesters 1,2,3)	**2. Period of study (semesters 5,6,8)**
Stress analysis (4 hrs)	Control theory (12 hrs)
Engineering mechanics (14 hrs)	Machine dynamics (4 hrs)
Physics (8 hrs)	Measurement theory (8 hrs)
Electrical engineering (4 hrs)	Finite Element Method (4 hrs)

It should be noted that similar intentions are pursued in the "Connected Curriculum Project" (CCP) in the USA (see: http://www.math.duke.edu/education/ccp/). But there, a suite of mathematical topics is offered with links to application cases whereas in our material we fully cover the application subject and there are links to necessary mathematics. Nonetheless, a sound structure of mathematics is presented by the author in the regular math lectures where also interconnections and modelling properties of mathematical concepts are emphasized.

3 Relearning material for control theory

The relearning material on "mathematics for control theory" is based on the lecture notes of the colleague teaching the subject. All mathematical topics and algorithms occurring in the scriptum are described and explained in the relearning material. For control theory I, the following areas are covered:

- Functions and their modelling properties
- Relations between functions (input <-> output functions)
- Differential equations (for describing the coupling between input and output)
- Matrices and determinants (Hurwitz determinant)
- Complex numbers
- Parameter representation of plane curves

For the subsequent lecture ("control theory II"), the following mathematical topics are needed:

- Linearization and normalization (could also be covered in I)
- Heaviside function and Dirac "function"
- Rational functions (poles, zeros, partial fractions)
- Laplace transformation
- Matrices and systems of linear differential equations

For each of the above-mentioned topics, the main definitions and results are stated as far as they are of importance in control theory. We use the notation of control theory such that notational confusion is avoided. It is clear that the material cannot be organized as usual in mathematics, i.e. starting with axioms and stating results by logical deduction. The mathematical "chunks" should be as self-contained as possible but certainly have to rely on a basic mathematical understanding of the user. The material should be neither too terse to retain comprehensibility nor so voluminous that students cannot go through it within a reasonable period of time and are hence rather intimidated than motivated. Therefore, the material contains definitions, main results and examples with variation possibilities but no proofs.

In order to illustrate how these principles have been realized, we give an outline of the contents of the worksheet on functions and their modelling properties. The worksheet is organized in three sections:

- *Definitions:* In this section, basic definitions for functions, continuity, and differentiability are stated and examples are defined and plotted. The notion of limit is assumed to be known.
- *Operations on functions:* Basic operations like translation and dilatation (in both directions x and t) are treated modelling amplification and delay.
- *Modelling behaviour with functions:* The kinds of behaviour to be modelled here include: jumps ("switch on", input of a system); damping, saturation, or -more general - asymptotic behaviour (output function); oscillation (with damping; input or output). Correspondingly, discontinuous functions, exponential functions, trigonometric functions and combinations thereof are treated here.

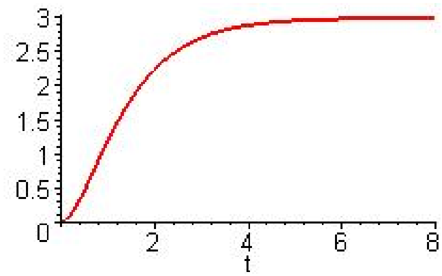
The printout of the whole worksheet comprises 19 pages (with lots of plots) and a student should be able to go through it in 2 hours (including experimentation and assignments, see below). With the information contained in the worksheet the student should be able to understand the mathematical model of operations on plants and their corresponding behaviour.

The reason for using computer algebra (Maple) as an implementation environment is that this way the material can be made interactive. Students can test the effect of changing parameters by modifying examples computed with Maple. For instance, the influence of the parameter in exponential damping can be made clear by variation and subsequent plot as the following Maple input lines and output show:

```
> saturation:=
>    t->a*(1+a2/(a1-a2)*exp(-a1*t)-a1/(a1-a2)*exp(-a2*t);
>    # definition of a saturation function
```

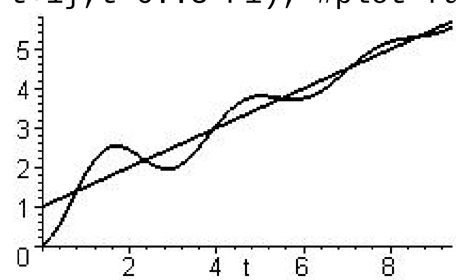
$$saturation := t \to a\left(1+\frac{a2\, e^{(-a1\, t)}}{a1-a2}-\frac{a1\, e^{(-a2\, t)}}{a1-a2}\right)$$

```
> a:=3: a1:=2: a2:=1:    # set values for parameters
> plot(saturation(t),t=0..8);
```



Such functions occur as output functions of so-called PT-2-plants and knowledge of the meaning and effect of parameter changes is important for understanding. One could as well examine the impact of parts of the function expression by plotting them isolated.
Students can also easily define and experiment with new input functions for differential equations to investigate how a system reacts on a certain input (for a more comprehensive treatment of the didactical potential of computer algebra systems as cognitive tools see [3]).
In order to provide students with some direction in their experimentation, we included assignments (with links to a solution worksheet) which serve to test and enhance understanding. For example, students are asked to construct functions modelling a certain behaviour or modify existing functions in order to get familiar with operations on functions. The computer algebra system is essential for providing such an experimental environment without any additional programming effort. For illustration purposes, we provide the following example (ambiguities are clear from the surrounding text):
Assignment: Construct and plot a function which starts at 0 with the value 0 and then oscillates damped-harmonically (angular frequency of 2) around the graph of t->t/2+1.
Possible Solution:

```
> f:=t->1/2*t+1+exp(-0.2*t)*sin(2*t-Pi/2): # function
> plot({f(t),1/2*t+1},t=0..3*Pi); #plot function and asymp.
```



Again, the assignments should not be too large in order to retain motivation. All worksheets can be viewed at http://www.fbm.fh-aalen.de/Lernmaterialien/Lernmaterialien.html.
In order to make it more worthwhile for students to use the offering and invest time to get a basic familiarity with Maple, we added worksheets which are specific to control theory. This also gives them an incentive to use computer algebra for application-specific experimentation. For control theory I, the additional material consists of:

- a worksheet on all basic plants (P, PT1, PT2, I, IT1, PI, ...) which computes unit step response, frequency response, polar plot and Bode diagrams. Here again, parameter values can be changed and the effect can be explored. This is to be used in the accompanying lab to compare theoretical output and measurements.
- A worksheet for simple control loops as far as they are required in the accompanying laboratory. This includes procedures programmed in Maple (TM). One could also perform the computations step by step as is shown in [4].

Note that similar offerings are made in commercial learning programs for control theory (see [5]) or production programs like Matlab/Simulink (TM) but for using these one has to change the environment (input syntax, style) and there are costs.
For control theory II, there is additionally a tutored assignment environment for the following type of assignments:

- Serial connection of elementary plants: Computation of frequency response and asymptotes in Bode diagrams using those of the elementary parts.
- Simplification of block diagrams.
- Computation of transfer functions for controlled systems with reference input and disturbance signals.

The corresponding worksheets contain assignments accompanying the lecture, so they are of relevance for the final examination. The tutoring consists of providing checking procedures which can be used to check intermediary results and which also give hints on possible errors if the result is wrong (for more information see [6]).

4 Accompanying activities

Since not every student had previous experience with Maple, we offered two introductory (or refreshing) session on Maple using a suite of simple demo worksheets which were also handed out to students. When in the future - according to the concept outlined in section 2 - students have already used Maple intensively during the first period of study, such sessions will be obsolete. Moreover, additional demo sessions served to show students how to use the mathematical relearning material in a meaningful and efficient way, as was described in the previous section.
For semester 5 which takes "control theory I" with an accompanying lab, another demo session on the contents of the worksheet with elementary plants and how to use it in the lab was offered. During the lab when students are to compute the theoretical output functions using the worksheet and compare them with measured ones, tutorial help was also available.
For semester 6 which takes "control theory II", another demo session on using the tutored assignment environment was offered. Showing students how they can check their own intermediary results and set up and work on new assignments to improve their performance in the exam is important for motivation.
All demo sessions take place during the first four to five weeks of the semester. The material is available in the departmental PC lab (and on the web). Moreover, a restricted number of Maple student licenses was handed out for the period of the semester such that students can also work at home.

5 Evaluation

The pure relearning material was offered in the winter term 1999/2000 for semesters 5 and 6 of mechanical engineering students (approximately 20 students per semester). Participation in introductory and demo sessions was voluntary and amounted to about 50%. At the end of the term, a questionnaire gave some results on the real usage which were partly disappointing.
Students of semester 5 did not use the material very much. They were mainly occupied with construction and were not able or willing to spend time in order to get a deeper understanding of the mathematics of control theory. Since the combined written exam on control theory I and II takes place only at the end of semester 6, students concentrated their efforts on construction. Students of semester 6 were more interested in the material and used it mainly at the beginning of the term. But their motivation also

lacked since the pure mathematical topics were not directly usable for exam or exam preparation.

This experience led to extending the material with the application topics described in section 3. Integration with lab experiments in semester 5 makes it directly relevant for the control theory lecture and hence much more interesting even though the exam is "far away". For semester 6, the added assignment environment offers the opportunity to improve preparation for the final exam thus giving a good incentive to spend time on it. Current experience indicates that these application-specific additions increased the acceptance of the complete set of materials with students considerably.

6 Conclusion and future developments

In this paper we presented material for relearning mathematics needed in the lectures on control theory. The material is part of a larger project on interconnecting mathematics and mechanical engineering subjects. Our student population has normally a rather utilitarian attitude towards mathematics and is very practice-oriented. This has to be taken into account when generalizing the following observations on acceptance and usage of the material offered:

Observation 1: Offering purely mathematical relearning material to get a deeper insight in the mathematical modelling is not sufficient as an incentive, particularly when students are very occupied with other application subjects like construction. Therefore, we included additional application-specific material which had to be used in the accompanying lab. Integrating the relearning material with the application subject is necessary and gives students more motivation to spend time on it.

Observation 2: Not surprisingly, usability for exam preparation is a strong criterion for students when additional offerings are made. Therefore, motivation can be increased considerably by providing an assignment environment with tutorial help where assignments are of relevance for the exam.

Observation 3: Working with computer algebra systems requires additional effort to get familiar with commands which is not worthwhile for many students if it is just an "add-on" for one particular subject. Therefore, we will provide material for all application subjects needing many mathematical concepts. Thus, computer algebra systems should be accepted as "the new calculator" for computation, experimentation and visualization.

Future developments are carried out according to the concepts presented in section 2. Material for engineering mechanics and machine dynamics will be available for the winter term 2000/01. Tutorial help within the assignment environments according to the possibilities described in [6] will make the material more suitable for self-study.

References

1. Alpers, B. (1999). Combining hypertext and computer algebra to interconnect engineering subjects and mathematics, *Proc. 4. Int. Conf. on Technology in Math. Teaching (ICTMT-4)*, Plymouth

2. Glowalla, U., Häfele, G., Hasebrook, J., Rinck, M. and Fezzardi, G. (1992). Wiederlernen von Wissen, U. Glowalla and E. Schoop (Eds.): Hypertext und Multimedia: Neue Wege in der computerunterstützten Aus- und Weiterbildung, Springer: Berlin, 332-351

3. Hillel, J. (1993). Computer algebra systems as cognitive technologies, C. Keitel and K. Ruthven (Eds.): Learning from computers: Mathematics education and technology, Springer: Berlin-Heidelberg, 18-47

4. Tocci, C. and Adams, S. (1996): Applied Maple for engineers and scientists, Artech House: Boston, London

5. Jochheim, A., Gerke, M. and Laaser, W. (1999): Multimedia courseware for basic control theory, IEEE Int. Symp. on Computer-Aided Control System Design, Hawaii
6. Alpers, B. (2000). Using computeralgebra for rapid development of ITS components in engineering, C. Frasson et al. (Eds.): Proceedings of the ITS'2000, Montreal, Springer Lecture Notes in Computer Science, to appear