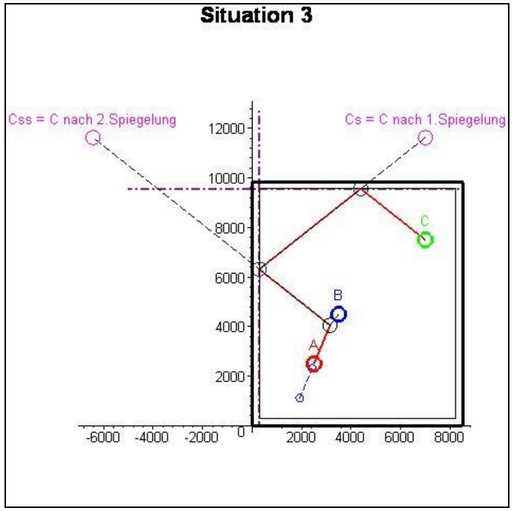
The Microworld "Billard"

(under construction)

"Billard" is a microworld for setting up carambolage configurations on a billard table and testing the effects of impacts. The microworld will support models of different complexity (with or without friction and energy loss at collision). A real billard machine has been constructed such that computed impacts (direction, starting velocity) can be tested on a real billard table. The simulation in Maple is currently developed.



Objects in the microworld will be the billard balls which can be positioned and impacted (i.e. provided with a certain velocity vector). Tasks within the microworld consist of constructing impacts for certain configurations such that collisions take place according to some carambolage rules. The microworld will give feedback on the motion curve of the balls as well as on angles and velocities before and after collisions such that the learner can correct or improve his constructions. Constrution of motion curves comprises geometric as well as algebraic modelling. Geometric operations include reflections, algebraic operations solving of non-linear equations. For the latter the learner can test with impacts in the microworld in order to find good starting values for the numerical solvers.
Feedback might look like the picture shown lefthand-side including geometrical constructions and position of balls at times of collision. Learners can start with a simple model without loss of energy where reflections can be used to construct impacts as is shown in the figure. They can then include friction and finally also the effects of loss of energy.

Literature & Contact

- Alpers, B.: Using CAS as Environments for Implementing Mathematical Microworlds, Int. J. of Computer Algebra in Math. Education 9 (2002), 177-204
- Gerny, M., Alpers. B.: Formula 1 - A Mathematical Microworld with CAS - Learning Opportunities and Real Usage, Int. J. of Computers for Math. Learning (to appear)

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Mathematical Application Microworlds (MAMs) with Links to Reality

What is a MAM?

Mathematical microworlds (Seymour Papert) are learning environments which contain computational objects and operations representing some sub-domain of mathematics. Manipulation of objects and application of operations serve to discover properties and to gain deeper understanding of the sub-domain.
If the objects and operations carry application meaning, i.e. they model a certain sector of the real world, we use the term "Mathematical Application Microworld (MAM)".
The links to reality in such a microworld can have different quality:

- the application meaning of objects can provide motivation for the learner and a background for posing meaningful questions and tasks in the microworld;
- the application can provide meaningful data input for setting up objects;
- it might be possible to test constructions made in the microworld in a real environment.

Which part of "modelling and application" is emphasised in a MAM?

A MAM provides already modelling means and hence facilitates the modelling process. There is a danger of trivialising this process on the one hand, but also an advantage of making the treatment of a part of reality feasible on the other hand. The objects available restrict the types of models which can be built, but – if designed properly – this leaves enough space for creative construction. The emphasis is on modelling in a certain model world.

What is the potential didactical value of a real model in addition to a simulation?

A simulation in normally somewhat restricted in the properties that are included in the model. Having in addition a real model provides new potential didactical opportunities:

- higher authenticity
- possibility of kinesthetic enactment (impact by queue in Billard, hand-driven motion)
- show restrictions of a simulation (e.g. jumps are not possible in reality); provide reason to think about what is possible, impossible and difficult to achieve in reality.
- use measurements from the real model as investigation or modelling material in simulations; e.g. investigate good "hand-driven" function on a toy racing circuit and try to construct similar function.
- give reason to go beyond the current simulation model (model progression, development of higher complexity) .

Further research must show whether the potential that has been identified can really be exploited to add value to the learning process over and above learning with purely computational environments.

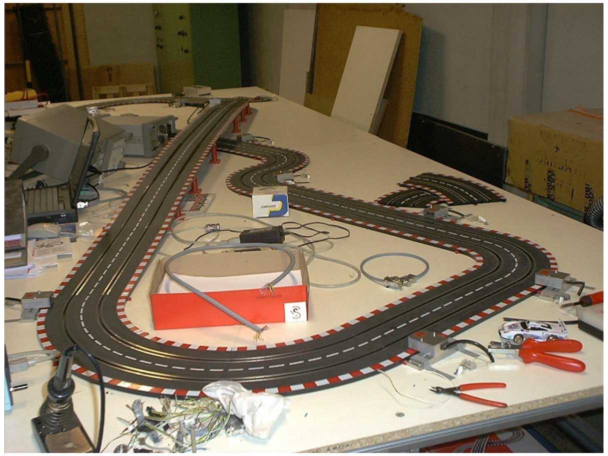
How can MAMs be implemented?

The computational part of a MAM can be implemented with reasonable effort in a Computer Algebra System (CAS) where standard mathematical objects and operations are already available and new application oriented objects and operations can be programmed.

This was demonstrated in the microworld "Formula 1" described below. The same approach is used in the microworld "Billard" which is under construction.

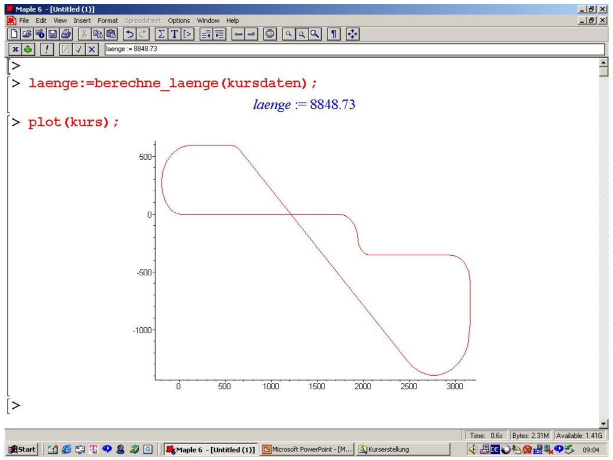
Considerably more effort is necessary to interconnect the simulation with a real model such that simulated situations can be performed in reality. Therefore, research is needed to check whether the potential value listed above can be realised such that the effort is worthwhile.

The Microworld "Formula 1"



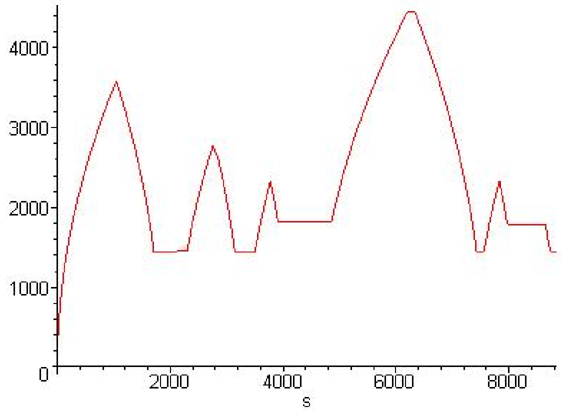
“Formula 1” is a microworld for constructing racing circuits and defining and investigating motion functions for such courses. One strong link to reality is a Carrera® toy course which provides realistic data (e.g. maximum velocity in a curve). The course is also used to compare computed laptimes of motion functions constructed in the microworld with realistic “hand-driven” laptimes. Currently, a microcomputer based device is built that allows to download a constructed function.

The computational microworld consists of a set of Maple ® objects and operations for course and motion construction.



A course is constructed as a list of course pieces where a piece can be a line segment or an arc segment. An operation "construct_course" sets up a piecewise defined curve in parameter representation which can be plotted as shown above. This gives feedback whether modelling has the desired results.

For constructing a motion function, the "native" Maple® objects for piecewise functions can be used (e.g. to construct velocity over time, v(t)). Microworld operations serve to compare the motion function with restriction functions like maximum velocity over distance. This gives feedback on violation of restrictions or potential of improvement where the ultimate goal is to achieve an optimal laptime. Below, the result of converting a learner provided v(t) function into a v(s) function is shown such that one can check easily whether the car would be too fast on certain parts of the course.



Mathematically, the microworld can be used to deepen understanding wrt. two topics: the geometry of line and arc segments and functions and their properties (continuity, growth etc.).

Exploiting the link to the real course which will be available in the future, we imagine to get data on real function to guide function construction as well as to download a desired function and measure the real effect.