

## Combining Hypertext and Computer algebra to interconnect Engineering Subjects and Mathematics

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### Abstract

Engineering students often have problems to understand the usage of mathematical concepts in engineering subjects. We developed learning software that makes the interconnections explicit and allows for switching between concepts and assignments. The software consists of a hypertext part and computer algebra worksheets (Maple).

The hypertext contains pages on all concepts of a one term class on stress analysis. Whenever mathematical concepts are used, there are links to mathematical concept pages such that a student can see the interconnections "on demand". For engineering concepts there are links to assignment worksheets which serve to deepen the understanding. Additional Maple procedures provide intelligent tutoring.

### 1. Introduction: Problems and Goals

In engineering education at university level, students often have problems to see the interconnection between mathematical concepts presented in math lectures by mathematicians and engineering concepts building upon math concepts presented by engineers. Sometimes simple notational differences hinder understanding, sometimes topics are used in applications before they have been dealt with in mathematics lectures.

Therefore, students often develop a rather negative attitude towards the usability of mathematical concepts in their area of interest. Many of them tend to restrict themselves to simply inserting values into formulae without really knowing what is modelled and what the model says about the influence of certain parameters. They are not inclined to experiment in order to obtain a deeper understanding of the models. Active learning is less achieved than passive "reception", and, as a consequence, the ability to apply models (let alone develop models) is weak. This certainly does not hold for all students. On the contrary, there are big differences within the student population which creates another problem: how can one differentiate in order to accommodate these differences without "losing" the weaker students or "boring" the strong ones.

The problems stated above can be partially addressed by adapting the lectures such that interconnections and modelling aspects are emphasized. Nonetheless, in each single subject the structure of the subject will still be dominating and there is no time to follow links and connections intensively. Therefore, by cooperation between lecturers only a certain amount of cohesion is achievable. Moreover, lectures are by their very nature restricted when it comes to differentiation and activation. It is the goal of the learning software<sup>1</sup> described in this paper to address just these problems. It consists of a documentary hypertext part (web pages) where connections between stress analysis and underlying mathematical concepts can be explored, and an activating assignment environment (computer algebra worksheets) which offers tutorial help and exploration capabilities that can be used according to the talent and knowledge of the student. Note that the software is meant to **accompany** the application course (here: stress analysis), not the math course. It does by no means substitute the lecture but should add value to it. So, the primary goals are to make interconnections explicit and to allow for switching between concepts and interactive exploration.

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Similar approaches have been pursued in other projects (e.g. the "Connected Curriculum Project" in the US, cf. [Winkelmann] for internet sites and discussion) but in our project the emphasis is not on math with some links to applications but on the application subject (i.e. stress analysis) with links to underlying math. As to the usage of computer algebra as cognitive tool and for intelligent tutoring, our approach is similar to the one in [Deeba/Gunawardena].

In the next section, we describe the structure of our program and give an outline of potential usage. Section 3 contains some remarks on implementation issues as far as adaptation is concerned. It is absolutely necessary to support students in using the software, so there were a couple of accompanying support activities which will be stated in section 4. Finally, we present the experience made in the summer term 99, discuss problems and further developments.

## 2. Structure and intended usage of the program

The learning program described below can be fully explored on the web server of the department of mechanical engineering of the Aalen University of Technology and Business: <http://www.fbm.fh-aalen.de> . Therefore, we only describe the main structural parts and design decisions without going into detail. More reasoning on the underlying concept which was worked out according to a methodology developed by [Kerres], can also be found in [Alpers].

The software consists of two main parts:

- A documentation part containing results
- An activation part containing assignments.

### Startseite

#### Festigkeitslehre I - Gliederung

- Spannungsdefinition
- Grundbelastungsarten
- Lagerungsarten
- Auflagerreaktionen
- Schnittprinzip der Festigkeitslehre
- Hooke'sches Gesetz für Normalspannungen (einachsig)
- Hooke'sches Gesetz für Schubspannungen (einachsig)
- Biegespannung  $\sigma_b$
- Querschnittswerte: Fläche, Schwerpunkt, Flächenmomente
- Satz von Steiner
- Satz über zusammengesetzte Querschnitte
- Umrechnung bei Drehung des Koordinatensystems
- Hauptträgheitsachsen und Hauptträgheitsmomente
- Symmetrische Querschnitte (Satz über das Zentrifugalmoment, Hauptträgheitsachsen)
- Mohr'scher Trägheitskreis
- Torsion von rotationssymmetrischen Querschnitten
- Querkraftschubspannung
- Verhältnis der Maximalwerte: Querkraftschubspannung zu Biegespannung bei Querkraft
- Einfluß des Querkraftschubs auf die Durchbiegung von Trägern: Schubverteilungszahl  $\kappa$
- Kerbspannung

Verzweigungen:

Gliederung Festigkeitslehre - Index - Mathematikthemen - Maple (Computeralgebra)

Startseite

Figure 1: List of contents (stress analysis)

The documentation part contains all concepts (and examples) of stress analysis which are taught in the respective lecture for mechanical engineers at the Aalen University for Technology and Business in a one term lecture. The list of contents is shown in figure 1. Note that this part does not comprise the whole exposition given in the lecture but only the results including some drawings to illustrate the topics. The list of topics is in the same order as they are taught in the lecture; it provides the red thread of the documentation part.

Whenever mathematical concepts are used in the description of an application topic, there are links to mathematical concept pages such that a student can look them up and see the interconnections "on demand". By caring for notational consistency, the usual problems can be avoided. Note that the mathematical part is not sequentially ordered and does not serve as a support for a mathematics lecture.

Umrechnung bei Drehung des Koordinatensystems

Gegeben:  $I_x, I_y, I_{xy}, I_{pol\ x,y}, \varphi$

Gesucht:  $I_u, I_v, I_{uv}, I_{pol\ u,v}$   
( Zeichnung zur Veranschaulichung, aus Platzgründen ausgeblendet)

Lösung:

$u, v$  = Schwerpunktskoordinaten der Teilfläche  $dA$  im  $u, v$ -Koordinatensystem .

Koordinatentransformation :

$$u = x \cdot \cos \varphi + y \cdot \sin \varphi$$

$$v = y \cdot \cos \varphi - x \cdot \sin \varphi$$

(Mathematik: -> Koordinatentransformation)

Eingesetzt in die Integrale ergibt sich bei Anwendung der Additionstheoreme von Sinus und Cosinus:

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cdot \cos 2\varphi - I_{xy} \cdot \sin 2\varphi$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cdot \cos 2\varphi + I_{xy} \cdot \sin 2\varphi$$

$$I_{uv} = \frac{I_x - I_y}{2} \cdot \sin 2\varphi + I_{xy} \cdot \cos 2\varphi$$

Man erhält also die Momente als Funktionen des Winkels  $\varphi$ : (Mathematik: -> Funktion,  
-> Additionstheoreme)

$I_u = I_u(\varphi), I_v = I_v(\varphi), I_{uv} = I_{uv}(\varphi).$

Figure 2: Example of links to mathematical topics

Figure 2 shows an example for links to mathematics within a description of an application topic. The topic under consideration deals - among others - with the change of moments of inertia caused by rotating the coordinate system. Here, several mathematical concepts are referred to:

- the transformation of coordinates
- the computation of moments using addition theorems for sine and cosine
- the usage of functions for describing the dependency between rotation angle and moments

These mathematical topics are described in the mathematical part of the hypertext pages. The exposition given there is mainly restricted to definitions, rough explanations and plausibility considerations, illustrations and examples. There are no proofs, nor is there any kind of axiomatic, logic derivation. So, this cannot be used for learning the contents but it should be enough to make clear to the engineering student, what is mathematically behind the concept. As already remarked by [Winkelmann], this way only "small portions" of mathematics can be brought back into memory. The portions must be essentially self-contained such that the student can go back to the application pages after a "short" mathematical break. It is still the task of the mathematics lecture to care for mathematical cohesion.

The hypertext pages on stress analysis and mathematical topics constitute the core of the documentation part. There are also pages containing tables with material data which can be used within assignments. Besides this, there are some pages containing general information on the aim and usage of the program. Another documentation part deals with computer algebra. Here, general information on computer algebra systems (CAS) can be found as well as a link to an introduction to Maple by Walz which is available in the web (<http://SunSITE.informatik.rwth-aachen.de/maple/maplev.html> ). In this part of the documentation one can also view all the Maple assignment worksheets as hypertext pages (automatically produced by Maple). So, just to get an overview of what is available one does not have to leave the hypertext documentation part.

The hypertext pages are linked according to the following principles: There is a starting page, from where one gets to the main parts described above. From each page it is possible to go back to the starting page as well as to the main sub parts. This can be seen, for example, in the contents page depicted in figure 1. From each page on a topic on stress analysis, one can go via a link to the "next concept"; recall that there is an ordering wrt. stress analysis concepts provided by the lecture. As described above, there are links from stress analysis pages to related mathematical concept pages. There are also links within the stress analysis part and within the mathematical part whenever terms are used which are described on another page. In order to look up a specific term, one can go to the index page which contains the respective links. Note that there are no links between worksheets and hypertext since these are handled by two different programs (cf. Section 3 below).

The documentation part contains a list of assignments. This list consists of links to Maple worksheets. A mouse click on such a link leads to loading Maple with the respective worksheet (this happens in the local version, in the internet version the worksheet is downloaded). For each concept in stress analysis there are one or more worksheets. They constitute the activating part of the learning environment.

Each worksheet has three sections:

- In the first section the assignment is described. Most descriptions also contain a drawing for clarification. Each assignment consists of a basic part which each student should work on, and an additional, more advanced part which should lead to experimentation and a deeper understanding of the influence of certain parameters.
- The second section serves as an environment for working on the assignment. An example for such a section is depicted in figure 3. First, variables to be used are reset (they might have had values resulting from former computations). Here, the student also sees exactly which names should be used to make tutorial help procedures possible. The latter are loaded next. Two kinds of support are offered to a student working on the assignments: First, for different stages of finding a solution verbal help and intermediate results are available. The level of detail varies. Sometimes, the student is asked to look up a certain concept or formula in the hypertext, sometimes a task-specific formula is given. This help is accessible via Maple links to sections in another help worksheet. Besides this, there are

Maple procedures which enable the student to check intermediate results and to get information on mistakes. This kind of "intelligent tutorial help" will be described in more detail below. The help should enable the student to go on in solving the problem in case he/she gets stuck. This is particularly important when a student works on the problem alone at home with no other kind of help available. As already stated in [Farinetti/Scarafiotti], this is very important for motivational reasons.

- The last section of the worksheet contains the solution but only the final results are given in order to prevent students from looking up the complete solution too early.

**Bearbeitung**

Verwenden Sie folgende Maple-Variablen.  
 > a:='a':b:='b':c:='c':d:='d':e:='e':h:='h':Fq:='Fq':  
 > read `C:\Lernhypertext\maple\berechne\_Querschnittswerte.mpl`:

Folgende Hilfen stehen bei der Bearbeitung zur Verfügung (nutzen Sie diese nur bei Bedarf!)  
Hilfe bei der Vorgehensweise; Hilfe zur Querkraftschubspannung;

Ferner können Sie Ihre Teilergebnisse mit folgenden Prozeduren überprüfen  
 (In der Sektion "Lösung" unten sind lediglich die Endergebnisse angegeben):  
 berechne\_statische\_momente(polygonzug);  
 berechne\_statische\_momente(polygonzug, punkt);  
 berechne\_traegheitsmomente(polygonzug);  
 berechne\_traegheitsmomente(polygonzug, punkt);

Figure 3: Second section of a Maple worksheet

Some tasks require symbolic computation, others need numerical computation or graphical depiction (e.g. position of principal axes). In particular for the symbolic part, a computer algebra program like Maple provides an adequate working environment. In the current version, students can use Maple in three different ways:

- The normal capabilities of Maple can be used for checking the correctness of symbolic "paper" computation. E.g., the "solve" command provides parameter dependent solutions of systems of equations. Since the systems needed in our setting are linear, there are no problems to decide whether there is no solution or Maple simply did not find one. In the experimentation part, Maple can be used to check the effect of parameter variation. Fixing all but one parameter and plotting the graph gives a good impression on the influence of this parameter. This is the typical usage of a CAS as a "cognitive tool" (cf. [Hillel], [Heugl et al.]). Yet, it is not just given to the students as a tool but within the framework of concrete assignments. [de Jong et al.] found out (for simulation tools) that students or pupils make a much more efficient use of a tool when it is not just a "playing ground" for vague experimentation but concrete problems have to be solved. An example for an assignment that guides experimentation is the task to find out how to choose the value of a certain quantity such that the centre of gravity lies in a certain area.
- Secondly, additional procedures serve to check intermediate results and to give hints on mistakes. For example, in certain assignments the student has to set up equations from equilibrium conditions. Such a symbolic system of equations can be checked for correctness and completeness by a so-called "check procedure". Moreover, the response also contains a statement on which equation is wrong such that the student has a higher chance to find his mistake. The procedures implemented so far are task-specific but with

more programming effort it would be possible to write such procedures for a generic type of assignments.

- Thirdly, there are "computation procedures" for computing certain data like the centre of gravity and moments of inertia. This can also be used for checking the correctness of intermediate results as is done by "check procedures", but besides check of results (symbolic or numeric), they also allow for higher level assignments of a synthetic or constructive nature. E.g., starting with a certain cross section area, students can be told to construct a cross section with certain properties concerning principal moments of inertia and area. They have to think about how a change in the input parameters influences the variables under considerations and check the validity of their hypotheses by computing them using the computation procedures.

In our check and computation procedures we make heavy use of the specific strength of computer algebra systems, i.e. their capability of symbolic data processing.

We anticipate the following three usage scenarios for our program: during the term accompanying the regular lecture; as a means to prepare for the exam; as a means to refresh knowledge for the subsequent lecture in the next term or for other lectures where the results are needed.

For the first scenario, accompanying the lecture, we envisage the following usage activities: students should look up concepts that were presented in the regular class and go to the underlying math concepts if required; they should then go to assignments on the computer algebra worksheets to deepen their understanding. When working on these assignments they can switch back to the documentation part to look up results in the application area or in mathematics. They should use help as needed such that implicitly differentiation is taken care of. They can check results using check procedures or general Maple capabilities and thus get stepwise feedback without obtaining the whole solution. The stronger students should work on the additional assignments and use Maple and computational procedures for experimentation. The average time period for working with the software should be about one hour per week.

In addition, the software can be used to prepare for the end term exam: Students can use the documentation part to refresh the results, and to have an overview of the structure of the application area. They can go over assignments again, and - in some cases - generate additional assignments for training purposes. Note that there is as yet no automatic assignment generation but in case of computation of cross-section values the existing computational procedures can be used for checking results for arbitrary polygons. So, the student can produce a polygon himself, work on this and check the results.

Finally, the program can be used to refresh parts of the subject or to look up formulae. The results learned in the stress analysis class will be particularly needed in the course on finite element methods in structural analysis, but also in other courses on machine elements and construction. It is not clear whether or not students are willing or able to invest more time and have also another look at the assignments. One could provide special refreshment assignments for such a purpose and start the class with performing those.

These usage scenarios take into account that the time schedule for engineering students is very tight (30 hrs of lectures per week). Therefore, additional offerings like our software will only be used if they see a clear advantage with respect to understanding, working on assignments, and passing exams. It would be non-realistic to expect the students to invest more additional work.

### 3. Implementation Issues

Implementation issues are important for the following reasons:

- First of all, the program should not require costly investments in additional programs.
- Secondly, the program should be extensible and adaptable such that it is interesting for other lecturers which can modify it to suit their lecture.

The documentation part, i.e. the hypertext, is implemented as web pages (HTML) including a lot of gif images for formulae and drawings (produced by word and image processing as well as CAD programs). Therefore, the two requirements stated above are fulfilled: the well-known browsers which are freely available, can be used for viewing the pages. Moreover, new pages can be added and linked into the structure and existing pages can be modified.

The activation part is implemented as Maple worksheets and Maple procedures. Clearly, a Maple student licence is required for using these CAS worksheets. For the duration of the project, the students are provided with such a license. The cost is not very high. It is clear that worksheets can be freely modified and also added to the current set. Since the procedures are also available as source code, they can be adapted as well.

### 4. Accompanying Activities

It is by no means sufficient to simply provide students with a learning program like ours and give no support afterwards. All but one of the students had private access to a PC (mostly their own one), so the computing infrastructure did not constitute a problem. As far as the handling of the program is concerned, the documentation and activation parts needed different treatment:

- The documentation part consists of hypertext in form of web pages which can be viewed with an internet browser. Students are quite familiar with internet browsing, so there was no necessity to give an introduction into the usage of hypertext and browsers.
- The activation part is implemented in form of Maple worksheets and none of the students had any experience with Maple or other CAS. Therefore, an introduction was necessary. For this, we set up several introductory worksheets which are also part of the learning program and accessible via hypertext links. We spent two to three regular lecture periods (90 minutes) on this. It is important to restrict oneself to the elementary features of Maple in order not to confuse the students. They do not use Maple all the time, so they easily forget specifics, and it is counter-productive wrt. motivation to show the real capabilities right at the beginning. Nevertheless, there were lots of questions during the term on Maple properties which were answered after the regular lectures.

Besides teaching the usage of Maple as a CAS, it is also important to show students how they can make use of the learning program. For this, three weeks after the beginning of the term a demonstration lecture was offered. Here, it was shown how one could go through the hypertext in a meaningful way, i.e. the usage scenarios described in the previous section were demonstrated. In the middle of the term, there was another demonstration session where especially the usage of tutorial help in Maple worksheets was shown. In particular, it was discussed how and when the check und computation procedures can be useful in the solution process.

After the final exam, the program was evaluated by the students in a questionnaire.

## 5. Evaluation and future perspectives

We tested the learning software with 10 first year students who began their studies in the summer term 1999. This is certainly not a significant number; in the winter term there will be about 60 beginners, so there will be an evaluation of higher statistical relevance. Nevertheless, the small number of students made possible a close contact and direct help.

Reeves distinguishes between several kinds of evaluation (cf. [Reeves]): Among those, the "formative evaluation" and the "effectiveness evaluation" apply to our setting. Whereas the former takes place during the production and/or the implementation of the program and suggests immediate improvements and changes, the latter looks at the outcome and compares it with the initial goals. As to formative evaluation, we talked to students about their usage and problems with the program and, as a result, offered additional demonstrations. So, we changed the accompanying activities but not the program itself. In order to perform effectiveness evaluation, we set up a questionnaire concerning the real usage of the program (duration, frequency, kind of usage, separately for the hypertext and computer algebra parts), and talked to the lecturer about impressions and exam results.

In the following we present some findings which show the dependancy of usage on the knowledge and prerequisites of the students. One should keep in mind that the student group consisted of only 10 persons.

- **General usage:** Most of the students made use of the program once in two weeks (about 7-8 times during the term) with an average duration of 30-45 minutes. They spent more time on the program during the middle and the end of the term which is not surprising since they needed some introduction during the first weeks. The students used Maple not only for the assignments in stress analysis but also for doing assignments in other subjects, so one positive result certainly is that students learned to make use of CAS.
- **Usage of hypertext:** The hypertext was much less used than anticipated. Few used the possibility to refresh the mathematics referred to on the stress analysis pages. The students stated that they were still familiar with the mathematics needed or the short presentation of the latter during the lecture on stress analysis was sufficient for them. This corresponds strongly to their marks in the final exams in mathematics as well as in stress analysis: The average marks were significantly better than in former classes. Therefore, only a test with weaker students will show whether or not the links to mathematics are helpful. Some of the students used the stress analysis pages for exam preparation and all but one for clarifying the structure of the subject.
- **Usage of Maple:** Most of the students used Maple as an environment for working on the assignments concerning cross-section characteristics like moments of inertia during the middle of the term, whereas the first assignments were not dealt with at all (note that they were not yet familiar with Maple right from the beginning and obviously, they were not motivated later on to go over the former assignments again). All of the students used Maple to check the results of their paper calculation. Since the final exam is done on paper, it is not surprising that students try to solve the problems on paper first. As to the tutorial help, students made only use of the computation procedures. They mainly did not need the other help facilities (help worksheets, special check procedures) since they had no major problems and checking their computation with general Maple facilities was sufficient.

It is interesting to note that none of the students worked on the additional assignments which were directed at stronger students although there were a lot of talented ones in the class. As a reason they stated that they spent the time on those subjects where they had more problems, i.e. physics and engineering mechanics.

Summing up, one can state that though activation was achieved, experimentation is still in its infancy.

- **General assessment by the students:** Two students found the program very useful, five useful, and one less useful. The majority of them wanted to have still more demonstrations. It was also the experience of the author that in particular for freshmen, it is quite important to show the usability and usefulness of the program during the whole term for different classes of assignments. All students thought that the program would be useful to refresh their knowledge at the beginning of the next term. Most of them stated that similar programs would be useful in subjects which are more problematic for them, i.e. physics and engineering mechanics.

From these findings as well as from conversation with students we draw the following conclusions, in particular concerning further improvements:

- The accompanying activities were the absolute minimum to make sure the students know what they can do with the program. More demo sessions would be helpful and motivating. Moreover, sessions where students use the software themselves and perform experiments should be offered such that they better see the potential for experimentation.
- Similar material should be offered particularly for those subjects where students have major problems. The students named engineering mechanics and physics, so these would be good candidates. If we had such material, learning Maple would become even more interesting for students since they could make use of it in several subjects. Besides this, if a suitable infrastructure is available, Maple should also be used for mathematics lectures and assignments.
- A well-founded evaluation will only be possible after the next term when there is a much broader spectrum of students available.
- Technically, the software could be improved by making the check procedures more generic (not only suitable for one specific assignment) and by creating assignments automatically. But before spending effort on this, it should be checked if students really make use of these kinds of offerings.

Finally, one should state that it would be non-realistic to expect everything from a learning program; personal contact and commitment, demonstration of interconnections in regular lectures are still important, in particular for the acceptance of an additional offering like our program.

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